This equation is easily simplified, and assumes the form

$$\frac{x^2}{a^2}=\frac{1+e^2}{3e^2}.$$

It follows at once that the least value of $3e^2$ is $1+e^2$, or the least value of e is $\frac{1}{2}\sqrt{2}$.

II. Solution by the PROPOSER.

If ϕ be the eccentric angle of any point of the ellipse $a^2y^2+b^2x^2=a^2b^2$, the equation to the corresponding circle of curvature is

$$x^2+y^2-(a^2-b^2)\left(\frac{2\cos^3\phi}{a}x-\frac{2\sin^3\varphi}{b}y\right)$$

$$+a^2(\cos^2\varphi-2\sin^2\varphi)-b^2(2\cos^2\varphi-\sin^2\varphi)=0.$$

This passing through the center, requires that

$$a^2(\cos^2\phi-2\sin^2\varphi)-b^2(2\cos^2\varphi-\sin^2\varphi)=0,$$

$$\text{or }\frac{a^2-b^2}{a^2}=e^2=\frac{1}{2-3\sin^2\varphi},$$

which is a minimum for $\phi=0$ or π; that is $e^2=\frac{1}{2}$.

Excellent solutions were received from *J. W. YOUNG*, *G. B. M. ZERR*, *J. SCHEFFER*, and *W. H. DRANE*.

CALCULUS.

98. Proposed by CHARLES CARROLL CROSS, Meredithville, Va.

Of the circumference of a fixed circle radius R rolls a circle radius r. Required the length of the curve described by a point on the circumference of the rolling circle; (1) when the circle rolls on the inside; (2) when the circle rolls on the outside of the circumference of the fixed circle.

Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; WALTER H. DRANE, A. M., Graduate Student, Harvard University, Cambridge, Mass.; J. SCHEFFER, A. M., Hagerstown, Md.; M. E. GRABER, Student, Heidelberg University, Tiffin, O.; and G. B. M. ZERR, A.M., Ph.D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

We have here the epicycloid and hypocycloid. The equation of the former is

$$x=(R+r)\cos\phi-r\cos\frac{R+r}{r}\phi,\text{ and }y=(R+r)\sin\phi-r\sin\frac{R+r}{r}\phi.$$

$$\therefore\frac{dx}{d\phi}=-(R+r)\sin\phi+(R+r)\sin\frac{R+r}{r}\phi.$$

$$\frac{dy}{d\phi}(R+r)\cos\phi-(R+r)\cos\frac{R+r}{r}\phi.$$

But $\dfrac{ds^2}{d\phi^2}=\dfrac{dx^2}{d\phi}+\dfrac{dy^2}{d\phi^2}=4(R+r)^2\sin^2\dfrac{R}{2r}\phi.$

$\therefore$ Length of curve between cusp and cusp

$$=2\int_0^{2+r/R}(R+r)\sin\frac{R}{2r}\phi=\frac{8r}{R}(R+r).$$

In the case of the hypocycloid we have in its equation only to put $-r$ in lieu of r, and by proceeding in the same way we obtain the length of the curve

$$=\frac{8r}{R}(R-r).$$

AVERAGE AND PROBABILITY.

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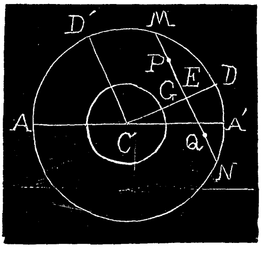
86. **Proposed by L. C. WALKER, Assistant in Mathematics in Leland Stanford, Jr., University. Palo Alto, Cal.**

Two points are taken at random in a circular annulus formed by two concentric circles. Find the chance that the straight line joining the points will not cut the inner variable circle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let P, Q be the two random points, MN the chord through P, Q



Let $AC=r$, $CE=w$, $MQ=x$, $PQ=y$, $\angle ACD'=\theta$. CG, the radius of the variable circle $=u$, $p=$required chance.

An element of the circle at Q is $dwdx$, at P $yd\theta dy$. The limits of u are 0 and r; of w, r and u; of x, $2\sqrt{(r^2-w^2)}$ and 0; of y, 0 and x and doubled; of θ, 0 and 2π.

$$\therefore\ p=\frac{\dfrac{2}{\pi^2r^4}\displaystyle\int_0^r\int_u^r\int_0^{2\sqrt{(r^2-w^2)}}\int_0^x\int_0^{2\pi}dudwdxydyd\theta}{\displaystyle\int_0^r du}$$

$$=\frac{4}{\pi r^5}\int_0^r\int_u^r\int_0^{2\sqrt{(r^2-w^2)}}\int_0^x dudwdxydy$$